Magnetic Catalysis of Chiral Symmetry Breaking in Gauge Theories.

V. P. Gusynin

Bogolyubov Institute for Theoretical Physics, Kiev-143, 03143 Ukraine

Abstract

Non-perturbative effect of the formation of a chiral symmetry breaking condensate $\langle \bar{\psi}\psi \rangle$ and of a dynamically generated fermion mass in QED in the presence of an external magnetic field is considered. The dynamical mass of a fermion (energy gap in the fermion spectrum) is shown to depend essentially nonanalytically on the renormalized coupling constant α in a strong magnetic field. Possible applications of this effect are discussed.

The dynamics of fermions in a strong external magnetic field has been attracting much attention during last years. Perhaps, the brightest example has been the discovery and theoretical explanation of the fractional Hall effect leading to the 1998 Nobel Prize award (see Nobel lectures by Laughlin, Stormer and Tsui in [1]) for "discovery that electrons acting together in strong magnetic fields can form new types of "particles", with charges that are fractions of electron charges ", as is said in press release of the Royal Swedish Academy of Sciencies. Thus, strong magnetic fields can drastically affect the ground state of a system leading to new types of excitations.

In this talk, I will describe one more phenomenon in an external magnetic field: dynamical breaking of chiral symmetry induced by such a field, hence the name magnetic catalysis.(The talk is based on a series of recent papers with V. Miransky and I. Shovkovy.) This effect has been established as a universal phenomenon in $2+1$ and $3+1$ dimensions: a constant magnetic field leads to the generation of a fermion dynamical mass at the weakest attractive interaction between fermions [2, 3, 4, 5]. The essence of this effect is that electrons behave effectively as (1+1)-dimensional ones when their energy is much less than the Landau gap $\sqrt{|eB|}$ (B is the magnitude of the magnetic field). The lowest Landau level (LLL) plays here the role similar to that of the Fermi surface in the BCS theory of superconductivity, leading to the dimensional reduction $D \rightarrow D - 2$ in the dynamics of fermion pairing in a magnetic field and to the formation of a chiral condensate at weak coupling. The effect may have interesting applications in cosmology [2, 6] and in condensed matter physics [7], as will be discussed below.

The effect of magnetic catalysis was studied in Nambu-Jona-Lasino (NJL) models in 2+1 [2, 8] and 3+1 dimensions [3, 9], it was extended to the case of external non-abelian chromo-magnetic fields [10], finite temperatures [11] and chemical potential [12, 13], curved space-time [14], confirming the universality of the phenomenon. In particular, this phenomenon was considered in $(3 + 1)$-dimensional QED [4, 5, 15, 16, 17, 18].

We emphasize that we will consider the conventional, weak coupling, phase of QED since the dynamics of the LLL is long-range (infrared), and the QED coupling constant is weak

in the infrared region, therefore, the treatment of the nonperturbative dynamics is reliable there. Note that chiral symmetry breaking is not manifested in the weak coupling phase of QED in the absense of a magnetic field, even if it is treated nonperturbatively [19]. We will show that a constant magnetic field B changes the situation drastically, namely, it leads to dynamical chiral symmetry breaking in QED for any arbitrary weak interaction.

The Lagrangian density of QED in a magnetic field is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left[\bar{\psi}, (i\gamma^\mu D_\mu - m_0)\psi\right],\tag{1}$$

where the covariant derivative D_μ is

$$D_\mu = \partial_\mu - ie(A_\mu^{ext} + A_\mu), \qquad A_\mu^{ext} = \left(0, -\frac{B}{2}x_2, \frac{B}{2}x_1, 0\right),\tag{2}$$

i.e. we use the so-called symmetric gauge for A_μ^{ext}. Besides the Dirac index, the fermion field carries an additional flavor index $a = 1, 2, \ldots, N$. When the bare mass $m_0 = 0$ the Lagrangian density (1) is invariant under the chiral $SU_L(N) \times SU_R(N) \times U_V(1)$ symmetry (we will not discuss the dynamics related to the anomalous symmetry $U_A(1)$).

We consider first the problem of free relativistic fermions in a magnetic field in $3+1$ dimensions and compare it with the same problem in $2+1$ dimensions. We will see that the roots of the fact that a magnetic field is a catalyst of chiral symmetry breaking are actually in this dynamics.

The energy spectrum of fermions in a constant magnetic field is:

$$E_n(p_3) = \pm\sqrt{m_0^2 + 2|eB|n + p_3^2}, \quad n = 0, 1, 2 \ldots\tag{3}$$

(the Landau levels). Each Landau level is infinitely degenerate. As the fermion mass m_0 goes to zero, there is no energy gap between the vacuum and the lowest Landau level with $n = 0$. The density of states of fermions on the energy surface with $E_0 = 0$ is given by

$$\nu_0 = \frac{|eB|N}{4\pi^2}, \quad 3+1 \quad \text{dimensions, and} \quad \nu_0 = \frac{|eB|N}{2\pi}, \quad 2+1 \quad \text{dimensions.}\tag{4}$$

The dynamics of the LLL plays the crucial role in catalyzing spontaneous breaking of chiral symmetry. In particular, the density ν_0 plays the same role here as the density of states on the Fermi surface ν_F in the theory of superconductivity. The next important point is that the dynamics of the LLL is essentially $(1+1)$-dimensional. Indeed, let us consider the fermion propagator in a magnetic field which was calculated by Schwinger [20] long ago and has the following form (in the chosen gauge):

$$S(x, y) = \exp\left(\frac{ie}{2}(x-y)^\mu A_\mu^{ext}(x+y)\right)\tilde{S}(x-y),\tag{5}$$

where the Fourier transform of $\tilde{S}$ is

$$\tilde{S}(p) = \int_0^\infty ds \exp\left[is\left(p_0^2 - p_3^2 - p_\perp^2\frac{\tan(eBs)}{eBs} - m_0^2\right)\right]$$
$$\cdot \left[(p^0\gamma^0 - p^3\gamma^3 + m_0)(1 + \gamma^1\gamma^2\tan(eBs)) - p_\perp\gamma_\perp(1 + \tan^2(eBs))\right].\tag{6}$$

Here $p_\perp = (p_1, p_2)$, $\gamma_\perp = (\gamma_1, \gamma_2)$ (to get an expression in $2 + 1$ dimensions we should put $p_3 = 0$ in (6)). The propagator $\tilde{S}(p)$ can be decomposed over the Landau level poles as follows [21] :

$$\tilde{S}(p) = i \exp\left(-\frac{p_\perp^2}{|eB|}\right) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(eB, p)}{p_0^2 - p_3^2 - m_0^2 - 2|eB|n} \tag{7}$$

with

$$
\begin{aligned}
D_n(eB, p) &= (\hat{p}_\parallel + m_0)\left[(1 - i\gamma^1\gamma^2\mathrm{sign}(eB))L_n\left(2\frac{p_\perp^2}{|eB|}\right)\right. \\
&\quad - \left.(1 + i\gamma^1\gamma^2\mathrm{sign}(eB))L_{n-1}\left(2\frac{p_\perp^2}{|eB|}\right)\right] + 4\vec{p}_\perp\vec{\gamma}_\perp L_{n-1}^1\left(2\frac{p_\perp^2}{|eB|}\right),
\end{aligned}
$$

where $L_n(x)$ are the generalized Laguerre polynomials ($L_n \equiv L_n^0$, $L_{-1}^\alpha(x) = 0$). Eq.(7) implies that at $p_\parallel^2, m_0^2 \ll \sqrt{|eB|}$, the LLL with $n = 0$ dominates and we can write

$$\tilde{S}(p) \simeq 2i \exp\left(-\frac{p_\perp^2}{|eB|}\right)\frac{\hat{p}_\parallel + m_0}{p_\parallel^2 - m_0^2}O^{(-)}, \tag{8}$$

where $\hat{p}_\parallel = p^0\gamma^0 - p^3\gamma^3$ and $\hat{p}_\parallel^2 = (p^0)^2 - (p^3)^2$. The matrix $O^{(-)} \equiv (1 - i\gamma^1\gamma^2\mathrm{sign}(eB))/2$ is the projection operator on the fermion states with the spin polarized along the magnetic field. This point and Eq. (8) clearly demonstrate the $(1+1)$-dimensional character of the dynamics of fermions in the LLL. This property is preserved also in the case when the fermion mass is generated dynamically. Since at $m_0^2, p_\parallel^2, p_\perp^2 \ll |eB|$ the LLL pole dominates in the fermion propagator, one concludes that the dimensional reduction ($D \to D - 2$) takes place for the infrared dynamics in a strong ($|eB| >> m_0^2$) magnetic field. Such a dimensional reduction reflects the fact that the motion of charged particles is restricted in directions perpendicular to the magnetic field.

Let us first calculate the chiral condensate in $2 + 1$ dimensions for free four-component fermions:

$$
\begin{aligned}
\langle 0|\bar{\psi}\psi|0\rangle &= -\lim_{x \to y} \mathrm{tr}S(x, y) = -\frac{i}{(2\pi)^3}\mathrm{tr}\int d^3p\, \tilde{S}_E(p) \\
&= -\frac{4m_0 N}{(2\pi)^3}\int d^3p \int_{1/\Lambda^2}^{\infty} ds\, \exp\left[-s\left(m_0^2 + p_3^2 + \vec{p}_\perp^2\frac{\tanh(eBs)}{eBs}\right)\right] \\
&= -m_0\frac{|eB|N}{2\pi^{3/2}}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s^{1/2}}e^{-sm_0^2}\coth(|eBs|) \to -\frac{|eB|N}{2\pi}, \quad m_0 \to 0,
\end{aligned} \tag{9}
$$

where Λ is an ultraviolet cutoff in Euclidean space and, for concretness, we consider $m_0 \geq 0$. Thus, as $m_0 \to 0$, the condensate $\langle 0|\bar{\psi}\psi|0\rangle$ remains non-zero due to the LLL. Note that the expression (9) is nothing else as the Banks-Casher formula relating the fermion condensate to the level density of the Dirac operator at zero eigenvalue [22]. The appearance of the condensate in the chiral (flavor) limit, $m_0 \to 0$, signals the spontaneous breakdown of the chiral (flavor) symmetry even for free fermions in a magnetic field at $D = 2 + 1$ [2].

Repeating the same calculation of the chiral condensate in $3 + 1$ dimensions we would get

$$\langle 0|\bar{\psi}\psi|0\rangle \simeq -m_0\frac{|eB|N}{4\pi^2}\left(\ln\frac{\Lambda^2}{m_0^2} + O(1)\right), \qquad m_0 \to 0, \tag{10}$$

i.e. the condensate is zero and there is no chiral symmetry breaking. Note, however, the appearance of logarithmic singularity in (10) due to the LLL dynamics. As we will see below, switching on even a weak attraction between fermions leads to the formation of chiral condensate in $(3 + 1)$-dimensional case.

The above consideration suggests that there is a universal mechanism for enhancing the generation of fermion masses by a strong magnetic field: the fermion pairing takes place essentially for fermions at the LLL and this pairing dynamics is $(1 + 1)$-dimensional in the infrared region. This is the main reason why in a magnetic field spontaneous chiral symmetry breaking takes place even at the weakest attractive interaction between fermions in $3 + 1$ dimensions [3, 4, 5].

Now we shall consider QED in $3 + 1$ dimensions whose Lagrangian is given by Eq. (1). To study chiral symmetry breaking one has to solve the Schwinger-Dyson (SD) equation for the dynamical fermion mass. The SD equation for the fermion propagator $G(x, y)$ in an external field has the form

$$G^{-1}(x, y) = S^{-1}(x, y) + \Sigma(x, y), \tag{11}$$

$$\Sigma(x, y) = 4\pi\alpha\gamma^\mu \int G(x, z)\Gamma^\nu(z, y, z')\mathcal{D}_{\nu\mu}(z', x)d^4zd^4z'. \tag{12}$$

Here $S(x, y)$ is the bare fermion propagator (5) in the external field A_μ^{ext}, $\Sigma(x, y)$ is the fermion mass operator, and $\mathcal{D}_{\mu\nu}(x, y)$, $\Gamma^\nu(x, y, z)$ are the full photon propagator and the full amputated vertex. The full photon propagator satisfies the equations

$$\mathcal{D}_{\mu\nu}^{-1}(x, y) = D_{\mu\nu}^{-1}(x - y) + \Pi_{\mu\nu}(x, y), \tag{13}$$

$$\Pi_{\mu\nu}(x, y) = -4\pi\alpha\mathrm{tr}\gamma_\mu \int d^4ud^4zG(x, u)\Gamma_\nu(u, z, y)G(z, x), \tag{14}$$

where $D_{\mu\nu}(x - y)$ is the free photon propagator and $\Pi_{\mu\nu}(x, y)$ is the polarization operator.

It is not difficult to show directly from the SD equations (11), (12), (13) and (14) that substitutions

$$G(x, y) = \exp\left(iex^\mu A_\mu^{ext}(y)\right)\tilde{G}(x - y), \tag{15}$$

$$\Gamma(x, y, z) = \exp\left(iex^\mu A_\mu^{ext}(y)\right)\tilde{\Gamma}(x - z, y - z), \tag{16}$$

$$\mathcal{D}_{\mu\nu}(x, y) = \tilde{\mathcal{D}}_{\mu\nu}(x - y), \tag{17}$$

$$\Pi_{\mu\nu}(x, y) = \tilde{\Pi}_{\mu\nu}(x - y) \tag{18}$$

lead to equations for translation invariant parts of Green's functions. In other words, in a constant magnetic field, the Schwinger phase is universal for Green's functions containing

4

one fermion field, one antifermion field, and any number of photon fields, and the full photon propagator is translation invariant.

We solve the SD equation for the fermion propagator in the so-called ladder approximation when the full vertex and full photon propagator are replaced by their bare ones. We have

$$\tilde{G}(x) = \tilde{S}(x) - 4\pi\alpha \int d^4x_1 d^4y_1 e^{ixA(x_1)+ix_1A(y_1)} \tilde{S}(x-x_1)\gamma^\mu \tilde{G}(x_1-y_1)\gamma^\nu \tilde{G}(y_1)\mathcal{D}_{\mu\nu}(x_1-y_1), \quad (19)$$

where the shorthand $xA^{ext}(y)$ stands for $x^\mu A_\mu^{ext}(y)$.

First, let us show that the solution to the above equation, $\tilde{G}(x)$, allows the factorization of the dependence on the parallel and perpendicular coordinates,

$$\tilde{G}(x) = \frac{i}{2\pi l^2} \exp\left(-\frac{x_\perp^2}{4l^2}\right) g\left(x_\parallel\right) O^{(-)}. \quad (20)$$

Notice that this form for $\tilde{G}(x)$ is suggested by a similar expression for the bare propagator,

$$\tilde{S}(x) = \frac{i}{2\pi l^2} \exp\left(-\frac{x_\perp^2}{4l^2}\right) s\left(x_\parallel\right) O^{(-)}, \quad (21)$$

with

$$s\left(x_\parallel\right) = \int \frac{d^2k_\parallel}{(2\pi)^2} e^{-ik_\parallel x_\parallel} \frac{\hat{k}_\parallel + m}{k_\parallel^2 - m^2}, \quad (22)$$

taken in the LLL approximation (here $l = |eB|^{-1/2}$ is the magnetic length). Performing the integrations we arrive at

$$g\left(x_\parallel\right) = s\left(x_\parallel\right) + 4\pi\alpha \int \frac{d^4q}{(2\pi)^4} d^2x_1^\parallel d^2y_1^\parallel \exp\left(-\frac{(q_\perp l)^2}{2} - iq_\parallel(x_1^\parallel - y_1^\parallel)\right) s(x^\parallel - x_1^\parallel)$$
$$\times \quad \gamma_\parallel^\mu g(x_1^\parallel - y_1^\parallel)\gamma_\parallel^\nu g(y_1^\parallel)\mathcal{D}_{\mu\nu}\left(q_\parallel, q_\perp\right). \quad (23)$$

Regarding this equation, it is necessary to emphasize that the "perpendicular" components of the γ-matrices are absent in it. Indeed, because of the identity $O^{(-)}\gamma_\perp^\mu O^{(-)} = 0$, all those components are killed by the projection operators coming from the fermion propagators. By switching to the momentum space, we obtain

$$g^{-1}\left(p_\parallel\right) = s^{-1}\left(p_\parallel\right) - 4\pi\alpha \int \frac{d^4q}{(2\pi)^4} \exp\left(-\frac{(q_\perp l)^2}{2}\right) \gamma_\parallel^\mu g(p_\parallel - q_\parallel)\gamma_\parallel^\nu \mathcal{D}_{\mu\nu}\left(q_\parallel, q_\perp\right). \quad (24)$$

The general solution to this equation is given by the ansatz,

$$g\left(p_\parallel\right) = \frac{A\hat{p}_\parallel + B}{A^2 p_\parallel^2 - B^2}, \quad (25)$$

5

where A and B are functions of $p_\parallel^2$. Making use of this as well as of the explicit form of the photon propagator in the Feynman gauge, we get that the function $A = 1$ while for the mass function we get the following integral equation

$$B(p^2) = m_0 + \frac{\alpha}{2\pi^2} \int \frac{d^2q B(q^2)}{q^2 + B^2(q^2)} \int\limits_0^\infty \frac{dx \exp(-xl^2/2)}{x + (q-p)^2} \tag{26}$$

(henceforth we will omit the symbol $\parallel$ from p and q). Thus the SD equation has been reduced to a two–dimensional integral equation. Of course, this fact reflects the two–dimensional character of the dynamics of electrons from LLL.

Analytical and numerical analysis of this equation were performed in [5] for the case $m_0 = 0$ and in [18] for nonzero bare mass. The numerical analysis showed that the so called linearized approximation, with $B(q^2)$ replaced by the total mass $m_{\text{tot}} \equiv B(0)$ in the denominator of Eq. (26), is an excellent approximation. Then we get

$$B(p^2) = m_0 + \frac{\alpha}{2\pi^2} \int \frac{d^2q B(q^2)}{q^2 + m_{tot}^2} \int\limits_0^\infty \frac{dx \exp(-xl^2/2)}{x + (q-p)^2}. \tag{27}$$

As was shown in [5] (see Appendix C), in the case of weak coupling α and for $m_0 = 0$, the function $B(p)$ remains almost constant in the range of momenta $0 < p^2 \lesssim 1/l^2$ and decays like $1/p^2$ outside that region. To get an estimate for $m_{dyn} \equiv B(0)$ at $\alpha << 1$, we set the external momentum to be zero and notice that the main contribution in the integral on the right hand side of Eq.(27) is formed in the infrared region with $q^2 \lesssim 1/l^2$. The latter validates in its turn the substitution $B(q) \rightarrow B(0)$ in the integrand of (26), and we finally come to the following gap equation:

$$B(0) \simeq \frac{\alpha}{2\pi^2} B(0) \int \frac{d^2q}{q^2 + m_{dyn}^2} \int\limits_0^\infty \frac{dx \exp(-l^2x/2)}{q^2 + x}, \tag{28}$$

which gives the expression for the dynamical fermion mass (energy gap in the fermion spectrum):

$$m_{dyn} \simeq C\sqrt{eB} \exp\left[-\sqrt{\frac{\pi}{\alpha}}\right], \tag{29}$$

where the constant C is of order one and $\alpha = e^2/4\pi$ is the renormalized coupling constant related to the scale $\sqrt{eB}$. The exponential factor in m_{dyn} displays the nonperturbative nature of this result. It can be shown also that the expression (29) for the dynamical mass is gauge invariant [4].

A more accurate analysis [5], which takes into account the momentum dependence of the mass function, leads to the result

$$m_{dyn} \simeq C\sqrt{|eB|} \exp\left[-\frac{\pi}{2}\sqrt{\frac{\pi}{2\alpha}}\right]. \tag{30}$$

The ratio of the powers of this exponent and that in Eq.(29) is $\pi/2\sqrt{2} \simeq 1.1$, thus the approximation used above is rather reliable.

We note that m_{dyn} has rather unusual $1/\sqrt{\alpha}$ behavior of the exponents in (29) and (30). Similar dependence was found recently in QCD for a quark gap arising at high densities (color superconductivity) [23]. The reason for such a behavior in both cases is the same: the presence of long-range interactions.

To study chiral symmetry breaking in an external field at nonzero temperature we use the imaginary time formalism. Now the analogue of the equation (27) (with $m_0 = 0$ and the replacement $m_{dyn} \to m^2(T)$ in the denominator) reads

$$B(\omega_{n'}, p) = \frac{\alpha}{\pi} T \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk B(\omega_n, k)}{\omega_n^2 + k^2 + m^2(T)} \int_0^{\infty} \frac{dx \exp(-l^2 x/2)}{(\omega_n - \omega_{n'})^2 + (k-p)^2 + x}, \quad (31)$$

where $\omega_n = \pi T(2n+1)$ are Matsubara frequencies.

If we now take $n' = 0, p = 0$ in the left hand side of Eq.(31) and put $B(\omega_n, k) \approx B(\omega_0, 0) = const$ in the integrand, we come to the equation

$$1 = \frac{\alpha}{\pi} T \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk}{\omega_n^2 + k^2 + m^2(T)} \int_0^{\infty} \frac{dx \exp(-l^2 x/2)}{(\omega_n - \omega_0)^2 + k^2 + x}. \quad (32)$$

The equation for the critical temperature is obtained putting $m(T_c) = 0$ and this determines the critical temperature [11]

$$T_c \approx \sqrt{|eB|} \exp\left[-\sqrt{\frac{\pi}{\alpha}}\right] \approx m_{dyn}(T=0), \quad (33)$$

where m_{dyn} is given by (29). The relationship $T_c \approx m_{dyn}$ between the critical temperature and the zero temperature fermion mass was obtained also in NJL model in (2+1)- and (3+1)-dimensions ([2] and second paper in Ref.[10]). The constant C, in the relation $T_c = C m_{dyn}$, is of order one and can be calculated numerically. We note that the photon thermal mass, which is of the order of $\sqrt{\alpha} T$ [24], cannot change our result for the critical temperature.

Taking into account the non-zero bare electron mass we come to the equation for the total mass m:

$$m \cos\left(\sqrt{\frac{\alpha}{2\pi}} \log \frac{|eB|}{m^2}\right) = m_0. \quad (34)$$

It can be shown [18] that the itterative solution of last equation reproduces all leading double logarithmic terms in perturbation theory:

$$m = m_0 \left[1 + \frac{\alpha}{4\pi} \log^2 \frac{|eB|}{m_0^2} + \frac{5}{24} \left(\frac{\alpha}{2\pi} \log^2 \frac{|eB|}{m_0^2}\right)^2 + \frac{61}{720} \left(\frac{\alpha}{2\pi} \log^2 \frac{|eB|}{m_0^2}\right)^3 + \cdots\right] \quad (35)$$

From Eq.(34) we can estimate the dynamical mass due to a magnetic field. For fields of the order of $\sim 10^{14} G$ which are realized on surfaces of neutron stars we get $(m - m_0)/m \sim 10\%$.

In the real QED the expansion parameter $\eta \equiv \frac{\alpha}{2\pi} \log^2(|eB|/m_0^2) \sim 1$ in (35) explores the transition between the perturbative regime $\eta \ll 1$ and the nonperturbative massless QED regime $\eta \gg 1$. The value of the parameter $\eta \simeq 1$ is reached at fields of the order $\sim 10^{26} G$. We recall that strong magnetic fields ($B \sim 10^{24} G$) might have been generated during the electroweak phase transition [25]. It has been speculated in Refs.[4, 5] that the character of electroweak phase transition could be affected by a generation of a dynamical electron mass under such strong fields. The nonperturbative regime becomes prevailing over the perturbative one for values of η of the order of 2.35 [18] what corresponds to magnetic fields $\sim 10^{32} G$. Ambjørn and Olesen [26] have claimed that even larger fields, $\sim 10^{33} G$, would be necessary at early stages of the Universe to explain the observed large-scale galactic magnetic fields.

Since the induced fermion dynamical mass contains an exponential factor (see (29), (30)) it is quite small at all reasonable values of the coupling α, therefore, there are tiny chances to find implications of the magnetic catalysis phenomenon in real experiments. However, it was shown recently [27] that the Yukawa coupling and scalar-scalar interaction can considerably enhance the fermion dynamical mass (according to [27] the dynamical mass is estimated to be $m_{dyn} \simeq 0.6\sqrt{|eB|}$). The most immediate physical implication would be then in the electroweak theory.

Another interesting application of the magnetic catalysis phenomenon is found in $(2 + 1)$-dimensional condensed matter systems [7], given the suggestions that high-temperature superconductors can be described effectively by $(2 + 1)$ relativistic field theories like NJL or QED (the relativistic (Dirac) nature of the fermion fields is related to the fact that they describe the quasi-particle excitations about the nodes of a d-wave superconducting gap). According to recent experiments [29], at temperatures significantly lower than T_c of superconductivity, the thermal conductivity, as a function of a magnetic field perpendicularly applied to the cuprate planes, displays a sharp break in its slope at a transition field B_κ, followed then by a plateaux region in which it ceases to change with increasing field. The critical temperature for appearance of the kink-like behavior scales with the magnetic field as $T_\kappa \sim \sqrt{e|B|}$. This phenomenon may indicate the opening of a second gap, at the nodes of the d-wave superconducting gap, that depends on the strength of the applied magnetic field [7, 29, 30]. Indeed, as we saw, in $(2 + 1)$-dimensional systems the chiral condensate appears even in absence of interaction between fermions. The dynamically generated fermion mass scales with a magnetic field like $m_{dyn} \sim \sqrt{e|B|}$ in $2 + 1$ NJL model [2], and $m_{dyn} \sim \alpha \log(\sqrt{|eB|}/\alpha)$ in QED3 [28]. The critical temperature for vanishing of the dynamical mass is determined by the dynamical mass at zero temperature (see Eq.(33)) and scales with a magnetic field in a way quite similar to the scaling law found in experiments.

In conclusion, we discuss very breifly the role of higher order radiative corrections in the magnetic catalysis problem. As was shown in Ref.[5], because of the $(1+1)$-dimensional form of the fermion propagator of the LLL fermions, there are relevant higher order contributions. In particular, considering this problem in the improved rainbow approximation (with the bare vertex in the Schwinger-Dyson equations for both the fermion propagator and the

polarization operator), it was shown that the fermion mass is given by Eq. (30) but with $\alpha \to \alpha/2$. Recently we have shown [31] that there exists a special (non-local) gauge in which the SD equations written in the improved rainbow approximation are reliable: in other words, in that gauge there exists a consistent truncation of the Schwinger-Dyson equations for this non-perturbative problem. The expression for m_{dyn} takes the following form,

$$m_{dyn} = \tilde{C}\sqrt{|eB|}F(\alpha)\exp\left[-\frac{\pi}{\alpha\ln(C_1/N\alpha)}\right],\qquad(36)$$

where N is the number of fermion flavors, $F(\alpha) \simeq (N\alpha)^{1/3}$, $C_1 \simeq 1.82$ and the constant $\tilde{C}$ is of order one.

Thus, the magnetic catalysis of chiral symmetry breaking in QED yields a (first, to the best of our knowledge) example in which there exists a consistent truncation of the Schwinger-Dyson equations in the problem of dynamical symmetry breaking in a (3+1)-dimensional gauge theory without fundamental scalar fields.

References

[1] R.B. Laughlin, Rev. Mod. Phys. 71, 863 (1999); H.L. Stormer, *ibid* 71, 875 (1999); D.C. Tsui, *ibid* 71, 891 (1999).

[2] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, Phys. Rev. Lett. 73, 3499 (1994); Phys. Rev. D52, 4718 (1995).

[3] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, Phys. Lett. B 349, 477 (1995).

[4] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, Phys. Rev. D 52, 4747 (1995).

[5] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, Nucl. Phys. B 462, 249 (1996).

[6] E.J. Ferrer and V. de la Incera, Int. J. Mod. Phys. A 14, 1 (1999).

[7] K. Farakos and N.E. Mavromatos, Int. J. Mod. Phys. B 12, 809 (1998); G.W. Semenoff, I.A. Shovkovy, and L.C.R. Wijewardhana, Mod. Phys. Lett. A 13, 1143 (1998); W.V. Liu, Nucl. Phys. B 556, 563 (1999).

[8] I.V. Krive and S.A. Naftulin, Phys. Rev. D 46, 2737 (1992); K.G. Klimenko, Z. Phys. C 54, 323 (1992).

[9] A.Yu. Babansky, E.V. Gorbar, and G.V. Shchepanyuk, Phys. Lett. B 419, 272 (1998); E. J. Ferrer, V. P. Gusynin, and V. de la Incera, Phys. Lett. B 455, 217 (1999).

[10] I.A. Shovkovy and V.M. Turkowski, Phys. Lett.B 367 213 (1996); D. Ebert and V.Ch. Zhukovsky, Mod. Phys. Lett. A 12, 2567 (1997); V.P. Gusynin, D.K. Hong and I.A. Shovkovy, Phys. Rev. D 57, 5230 (1998).

[11] D.–S. Lee, C.N. Leung, and Y.J. Ng, Phys. Rev. D 55, 6504 (1997); V.P. Gusynin and I.A. Shovkovy, Phys. Rev. D 56, 5251 (1997); V.P. Gusynin hep-ph/9709339.

[12] A.S. Vshivtsev, B.V. Magnitsky, V.Ch. Zhukovsky, and K.G. Klimenko, Phys. Part. Nucl. 29, 523 (1998); D. Ebert, K.G. Klimenko, M.A. Vdovichenko, and A.S. Vshivtsev, hep-ph/9905253.

[13] D.-S. Lee, C.N. Leung, and Y.J. Ng, Phys. Rev. D57, 5224 (1998).

[14] D.M. Gitman, S.D. Odintsov and Yu.I. Shil'nov, Phys. Rev. D 54, 2968 (1996).

[15] C.N. Leung, Y.J. Ng and A.W. Ackley, Phys. Rev. D 54, 4181 (1996).

[16] D.K. Hong, Phys. Rev. D 54, 7879 (1996); Phys. Rev. D 57, 3759 (1998).

[17] E.J. Ferrer and V. de la Incera, Phys. Rev. D 58, 065008 (1998).

[18] V.P. Gusynin and A.V. Smilga, Phys. Lett. B 450, 267 (1999).

[19] P.I. Fomin, V.P. Gusynin and V.A. Miransky, Phys. Lett. B 78, 136 (1978).

[20] J. Schwinger, Phys. Rev. 82, 664 (1951).

[21] A. Chodos, K. Everding and D.A. Owen, Phys. Rev. D 42, 2881 (1990).

[22] T. Banks and A. Casher, Nucl. Phys. B 169, 103 (1980).

[23] D.T. Son, Phys. Rev. D 59, 094019 (1999).

[24] H.A. Weldon, Phys. Rev. D 26, 1394 (1982).

[25] T. Vachaspati, Phys. Lett. B 265, 258 (1991).

[26] J. Ambjørn and P. Olesen hep-ph/9304220.

[27] E.J. Ferrer and V. de la Incera, hep-th/9910035.

[28] A.V. Shpagin, hep-ph/9611412.

[29] K. Krishana et al., Science 277, 83 (1997).

[30] R. Laughlin, Phys. Rev. Lett. 80, 4963 (1998).

[31] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, Phys. Rev. Lett. 83, 1291 (1999); Nucl. Phys. B 563, 361 (1999).